Econ 802

Final Exam

Greg Dow December 9, 2021

1. "If the output price rises for a competitive firm, its output quantity does not fall." Prove this using each of the following methods. Give clear verbal explanations for each step in your argument. Assume the firm has one output and n-1 inputs.

(a) The weak axiom of profit maximization (algebraic method).

(b) The properties of the profit function (duality method).

(c) Differentiation of the first order condition (implicit function method).

2. Asha has the indirect utility function $v(p_1 \ldots p_n, m) = \sum a_i \ln (a_i m/p_i)$ where $a_i > 0$ for $i = 1 \ldots n$ and $\sum a_i = 1$. Assume $(p_1 \ldots p_n) > 0$ and $m > 0$.

(a) The derivatives satisfy $\partial v/\partial p_i < 0$ for $i = 1 \ldots n$ and $\partial v/\partial m > 0$. Also, v depends only on the ratios m/p_i. Explain the economic intuition for each of these facts.

(b) In period 1, Asha faces prices p^1 and has income m^1. In period 2, she faces new prices p^2 and has just enough income m^2 to buy the same consumption bundle she bought in period 1. Does this imply $v(p^2, m^2) = v(p^1, m^1)$? Explain using a graph for the two-good case.

(c) Find the direct utility function $u(x_1 \ldots x_n)$ associated with $v(p_1 \ldots p_n, m)$.

3. Here are some miscellaneous questions.

(a) A profit-maximizing firm has production plans $y = (y_1, y_2)$ with $y_1 \leq 0$ and $y_2 \geq 0$. In periods $t = 1, 2$ the firm chooses the plans y^1 and y^2. Draw the smallest closed, convex, and monotonic production possibilities set YI that is consistent with these observations and justify your answer. If YI is the true PPS, does the firm's profit-maximization problem have a solution for every $(p_1, p_2) > 0$? Explain.

(b) A firm produces one output using two inputs. Its production function $f(x_1, x_2)$ has constant returns to scale. The input prices $(w_1, w_2) > 0$ are constants, and the firm minimizes cost for any output $y > 0$. Show that as y increases, the expansion path in (x_1, x_2) space is a ray from the origin.

(c) A consumer has the linear direct utility function $u = ax_1 + bx_2$ where $x_1 \geq 0, x_2 \geq 0$ and $a, b > 0$. The consumer faces prices $(p_1, p_2) > 0$ and has income $m > 0$. If you only knew the <u>indirect</u> utility function for this consumer and used Roy's identity to derive the Marshallian demands, would you usually get the right answer? Could there be situations where you would run into trouble using this method? Explain.

4. The nation of Variana has one consumer and two firms. The consumer has utility $u(x, y) = hx^{1/2} + y$ where $h > 0$ and $x \geq 0$ (y can have either sign). The consumer is endowed with $w > 0$ units of the y good and zero of the x good. The price of the x good is p and the price of the y good is one. Each firm has the same long run cost function $c(z) = a + bz^2$ with $a > 0, b > 0$, where $z \geq 0$ is the firm's output of the x good and $c(z)$ is the quantity of the y good the firm uses as an input.

(a) Show the supply function of an individual firm on a graph and explain it verbally. Then do the same thing for the market supply function when there are two firms. Finally, compute the consumer's Marshallian demand for good x.

(b) Suppose w is large enough to satisfy the input demands of the firms. Holding a and b fixed, identify all values of the parameter $h > 0$ such that there is some price p^* where supply and demand are equal in the market for the x good. Why is there no equilibrium price at other values of h? Explain using a graph.

(c) Think about this model as a case of general equilibrium with production. Suppose we tried to check for the existence of a Walrasian equilibrium using the properties of the aggregate excess demand function. Carefully discuss the issues that arise.

5. Consider a pure exchange economy with two consumers (A and B) and two goods (1 and 2). Consumption bundles are non-negative. A has the utility function $u_A = \min \{\alpha x_{A1}; \beta x_{A2}\}$ where $\alpha > 0, \beta > 0$. B has the utility function $u_B = x_{B1}^{1/2} x_{B2}^{1/2}$. A is endowed with $w_2 > 0$ units of good 2 and zero of good 1 while B is endowed with $w_1 > 0$ units of good 1 and zero of good 2. The prices are $(p_1, p_2) > 0$. In all parts of this question, assume $\alpha w_1 = \beta w_2$ (this will simplify a lot of things).

(a) Compute the Marshallian demands for good 1 for consumers A and B. Next, use the endowments to compute their incomes m_A and m_B. Then find the equilibrium price ratio p_1/p_2 by setting demand equal to supply for good 1.

(b) Draw an Edgeworth box showing the endowment point, the budget line, and the equilibrium point. Show the indifference curve passing through the equilibrium point for each consumer and explain why this allocation is Pareto efficient.

(c) The utility possibility frontier is linear here. Suppose a social planner would like to maximize $\theta_A u_A + \theta_B u_B$ where $\theta_A > 0$ and $\theta_B > 0$ are fixed weights. Discuss the possible outcomes using a graph in (u_A, u_B) space.